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KC-6149

B. E. - II (Sem. III) (Civil) Examination

November / December – 2012

Engineering Mathematics & Statistical Methods

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ B. E. - 2 (SEM. 3) (CIVIL)

Name of the Subject :

☛ ENGINEERING MATHEMATICS & STATISTICAL METHODS

☛ Subject Code No. : 6 1 4 9 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures on right indicate full marks of the questions.
(4) Assume suitable data if required.

1 (a) Attempt the following : **10**

(1) Show that $\overline{F} = 3y^4z^2i + 4x^3z^2j - 3x^2y^2k$ is solenoidal.

(2) Prove that $\frac{\beta(m, n+1)}{n} = \frac{\beta(m+1, n)}{m} = \frac{\beta(m, n)}{m+n}$

(3) Evaluate $\int_0^{\pi} \int_0^{\pi/2} \int_0^{1} r^2 \sin\theta \, dr \, d\theta \, d\phi$

(4) Change the order of $\int_0^{1} \int_{4g}^{4} e^{x^2} dA$

(5) Show that $grad\,(r) = \frac{\overline{r}}{r}$ where $\overline{r} = xi + yj + zk$.

(b) Attempt the following : **10**

(1) State and prove relation between Beta and Gamma function. **4**

(2) Evaluate $\int_0^{1} (x \log x)^3 \, dx$ **3**

(3) Evaluate $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+x}} \, dx$ **3**

2 Attempt any **three** : **12**

(1) Evaluate $\iint_R x^2y^2 dA$ where R is region in the first quadrant bounded by the circle $x^2 + y^2 = 1$.

(2) Find area lying between parabola $y = 4x - x^2$ and line $y = x$.

(3) Evaluate by changing order of integration $\int_0^a \int_{y^2/a}^{y} \frac{y\, dA}{(a-x)\sqrt{ax - y^2}}$

(4) Find volume of the solid bounded above by the paraboloid $z = 1 - x^2 - y^2$ and below by the xy-plane.

3 (a) Attempt any **two** : **8**

(1) A vector field is given by $\bar{F} = (\sin y)i + x(1 + \cos y)j$,

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ over the circular path $x^2 + y^2 = a^2$ and $z = 0$.

(2) Prove that $div\left(grad\ r^n\right) = n(n+1)r^{n-2}$.

(3) Show that $\bar{F} = \left(6xy + z^3\right)i + \left(3x^2 - z\right)j + \left(3xz^2 - y\right)k$ is irrotational and find its scalar potential.

(b) Attempt any **two** : **10**

(1) Using Stoke's theorem evaluate $\oint_C \left[(x+2y)dx + (x-z)dy + (y-z)dz\right]$ where C is the boundary of the triangle with vertices $(2, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 6)$.

(2) Verify Green's theorem in the plane for $\oint_C \left[\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy\right]$ where C is the boundary of the region defined by $x=0,\ y=0,\ x+y=1$.

(3) Evaluate $\iint_S \bar{F}\cdot d\bar{S}$ where $\bar{F}=4xzi+xyz^2j+3zk$ over the whole surface of the region above the xy-plane bounded by the cone $z^2=x^2+y^2$ and the plane $z=4$.

4 (a) Attempt the following :

(1) Solve $yq-xp=z$ **3**

(2) Write the solutions of $u_t=c^2u_{xx}$ **3**

(3) If $p=0\cdot03,\ n=100$, find $p(2)$ and $p(3)$ **4**

(b) If a string of length ℓ is released from rest, in the position $u(x,0)=\dfrac{4\lambda x(\ell-x)}{\ell^2}$ then find the displacement of the string at any point, at any instant. **8**

OR

(b) A bar of length $2m$ is fully insulated along its sides It is initially at a uniform temperature of $10°C$ and at $t=0$ the ends are plunged into ince and maintained at a temperature of $0°C$. Determine an expression for the temperature at a point p at a distance x from one end at any subsequent time t seconds after $t=0$. **8**

5 (a) Attempt any **two** : **6**

(i) $x^2p+y^2q=z^2$

(ii) $e^{-y}\cos xp-e^{-y}\sin xq-e^{-z}\sin x=0$

(iii) $pz-qz=z^2+(x+y)^2$

(b) Attempt any **two** : **10**

(1) The analysis of the employment structure of households in an urban area reveals that there are 2500 households of 4 members in each. Find the probability that a particular households of this size has 0, 2 and 4 employed residents.

(2) A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days

(i) on which neither car is used and

(ii) on which some demand is refused.

(3) The weight of 4000 students are found to be normally distributed with mean 50 kg and standard deviation 5 kg. Find the number of students with weight.

(i) less than 45 kg

(ii) between 45 kg to 60 kg.

(The S.N.V.Z. area between 0 and 1 is 0.3413 and between 0 and 2 is 0.4772)

6 (a) Fit a Binomial distribution to the following data and test the goodness of fit : **6**

x:	0	1	2	3	4	5	6	7	8	9	10
y:	6	20	28	12	8	6	0	0	0	0	0

(b) Attempt any **two** : **10**

(1) Fit a straight line for the following data :

x	50	70	100	120
y	12	15	21	25

(2) Calculate the correlation coefficient from :

x	1100	1200	1400	1500	1600	1700	1800	1900	2000
y	0.3	0.29	0.25	0.24	0.24	0.24	0.29	0.18	0.15

(3) If the probability that a man aged 60 will live to be 70 is 0.65, what is the probability that out of 10 men, now 60, at least 7 will live to be 70 ?